The Physics of the Human Ear

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In general, sensory organs can make a good topic for physics lectures.
Some of the advantages:

- It's interesting and more "real" to the students
- Students are familiar with the expected behavior and limits of their senses - they can finally use their intuition.
- Senses are complex, and often involve more than one physical principle. Often sense organs make a good "capstone" topic.
- Your school might already have some demos and displays - over in the biology department.

Lesson plans on the ear can combine several topics:

- pressure waves and resonators
- fluid mechanics
- lever mechanics and mechanical advantage
- phase and superposition

When possible, I ask the students to model the situation more than one way and then, based on their calculations, tell me what physical principle is being used by the organ. (See, *e.g.*, modeling the outer ear as a resonator.)

History

Most of what we know about the most basic workings of the ear is from von Helmholtz (1821 - 1894) and von Békésy (1961 Nobel Prize in Medicine/Physiology). Helmholtz in particular did an immense amount of work in all fields (including math and physics) and also did much pioneering work on the eye.

Biographies:
http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Helmholtz.html
http://www.nobel.se/medicine/laureates/1961/bekesy-bio.html

Units and Scales

Sound is a pressure wave. The SI unit of pressure is the Pascal (Pa).

$$1 \text{ Pa} = 1 \text{ N/m}^2 \qquad 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Sound waves are characterized by frequency. The SI unit of frequency is the Hertz (Hz).

$$1 \text{ cycle/s} = 1 \text{ Hz}$$

The common range of human hearing is from 20 - 20,000 Hz.

Loudness is measured in bels, or more commonly decibels (dB). The dB reflects the logarithmic sensitivity of the ear to sound. If the intensity of a wave (in power per unit area) is I, then the loudness in dB is

$$\beta = (10 \text{ dB}) \log\left(\frac{I}{I_o}\right)$$

The reference intensity is $I_o = 10^{-12} W/m^2$. This is defined as 0 dB. In the range of 2000 - 4000 Hz (where people are most sensitive) about 10% of the population can just hear a sound at 0 dB. A loudness of 120 dB is painful at all frequencies. You will often find this under the name *Weber-Fechtner Law*.

Example: Sound is a longitudinal pressure wave in air. What change in pressure and amplitude of oscillation (*i.e.*, what distance must a molecule be displaced) is needed in order to produce a 0 dB sound?

Assumed:	air density:	1.21 kg/m^3
	speed of sound in air:	340 m/s
	frequency:	3000 Hz (middle of range)

For a pressure wave, the maximum change in pressure is

$$\delta p_{\max} = (v\rho\omega)s_{\max}$$

where v is the speed, r is the density of the air, w is the angular frequency and s is the linear displacement of the molecule. (For a derivation of this, see, *e.g.*, Halliday, Resnick & Walker, Ch 18.)

The intensity of this wave is proportional to the square of the pressure change

$$I = \frac{1}{2}\rho v \omega^2 s_{\max}^2 = \frac{1}{2}\frac{\delta p_{\max}^2}{\rho v}$$

Solving this for pressure change gives us

$$\delta p_{\max} = \sqrt{2I\rho v} = \sqrt{2(10^{-12}\,\mathrm{W/m^2})(1.21\mathrm{kg/m^3})(340\mathrm{m/s})}$$

$$\delta p_{\max} = \sqrt{8.228\times 10^{-10}\,\mathrm{Pa}^2} = 2.89\times 10^{-5}\,\mathrm{Pa}$$

This is less than one billionth of standard atmospheric pressure!

Solving this for $s_{\max}$ gives us

$$s_{\max} = \sqrt{\frac{2I}{\rho v \omega^2}} = \sqrt{\frac{2(10^{-12}\,\mathrm{W/m^2})}{(1.21\mathrm{kg/m^3})(340\mathrm{m/s})(2\mathrm{p})^2(3000\mathrm{Hz})}}$$

$$s_{\max} = \sqrt{4.10\times 10^{-20}\,\mathrm{m}^2} = 2.03\times 10^{-10}\,\mathrm{m}$$

This is less than 1 atomic radius!

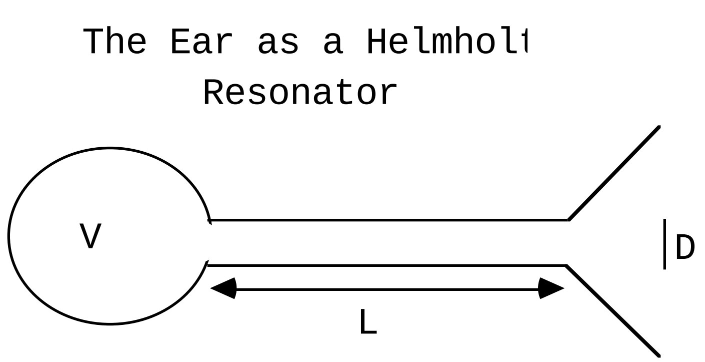
A similar calculation for the painful limit yields about 28 Pa for the maximum tolerable pressure change. This highlights the sensitivity of the ear to small fluctuations in pressure.

Anatomy of the Ear

Depending on your classroom situation, you might present this as
- a poster or model (check with your biology faculty - they might have one)
- a handout (see attached)
- an interactive web page
  http://hyperphysics.phy-astr.gsu.edu/hbase/sound/ear.html)
  http://library.thinkquest.org/19537/Ear2.html

Outer Ear

The outer ear acts primarily to gather in sound. It's directional shape also helps you to locate sound.



The ear canal is approximately 2.5 cm long and 0.35 cm in diameter. The middle ear has an approximate volume of 2 cm^3. Based on its physical layout, it is reasonable to model it as a Helmholtz resonator.

Using that model we would expect the ear to be most sensitive at a frequency of

$$f = \left(\frac{v_{sound}}{2\pi}\right)\sqrt{\frac{A}{VL}}$$

Using a speed of sound of 340 m/s, we would expect the ear to hear best at about 720 Hz. But all of our measurements indicate the best range is about 2000 - 4000 Hz.

So consider a pipe organ model, open at one end. We would expect a pipe to support a fundamental wavelength of 4L, so that gives us

$$f = \frac{v_{sound}}{\lambda} = \frac{v_{sound}}{4L}$$

This gives us a frequency of 3400 Hz. Thus the ear canal acts more like an open pipe than a resonator. The approximate amplification due to this resonance is 2x, meaning that the pressure at the far end of the ear canal is approximately twice that of the open end.

Middle Ear

The ear drum picks up the vibration in the ear canal and transmits it to the ossicles, three small bones in the middle ear. These bones, the *malleus* (hammer), the *incus* (anvil) and the *stapes* (stirrup), act as a lever system with a mechanical advantage of 2 or 3x. This amplification is frequency dependent, and again is most effective at 2000 - 4000 Hz.

In addition, the ossicles have an acoustic reflex to protect the ear. If a loud sound (approximately 85 dB or more) reaches the ear drum, the muscles of the inner ear twist, reducing the mechanical advantage and stiffening the ear drum. This lowers the effective intensity by about 20 dB. However, it takes approximately 150 ms for this reaction to occur, so sudden loud sounds (like a car backfire) can still damage your hearing. Incidentally, this flexibility is reduced with age (making your parents less appreciative of your music).

The end of the stirrup strikes the oval window at the beginning of the inner ear, behind which is a liquid called perilymph. This window has an area 20 to 30 times smaller than the ear drum. Keeping in mind that the ossicles are taking pressure (from the air in the eardrum) and amplifying that force (via a lever system) back to a pressure (in the perilymph), we can discuss the definition of pressure.

Recall that pressure is force over area. So our system has gone from a 2x amplification in the ear canal (for frequencies near resonance) through a mechanical advantage (2 to 3x) and an area change (A vs. $\frac{1}{20}A$ to $\frac{1}{30}A$). So, for the best possible amplification we expect

$$P_{oval} = \frac{F_{oval}}{A_{oval}} = \frac{3F_{drum}}{\frac{1}{30}A_{drum}} = 90P_{drum} = 90(2P_{drum}) = 180P_{air}$$

So the pressure has been increased passively anywhere between 80 to 180x. But keep in mind that the intensity of sound is proportional to the square of the pressure, which means an intensity amplification of 6400 to 32,000x!

The oval window is part of the cochlea, which is a tapered, liquid-filled organelle that wraps back on itself like a snail. Overall, one can imagine a single chamber, bent into a "V" shape, which is then rolled up from the tapered end. In such a small volume, the pressure wave in the perilymph is no longer simple and viscosity must be considered. The chamber is almost completely divided by the basilar membrane, The membrane is thicker and less taut at the beginning and becomes thinner and tighter at the far end. Here we can make an analogy with waves on a string, where the speed is dependent on both the density and tension of the string

$$v = C\sqrt{\frac{T}{\mu}} \Rightarrow \lambda = \frac{C}{f}\sqrt{\frac{T}{\mu}}$$

(*N.B.* the C is a dimensionless constant.) This means for a wave of a given frequency, there will be a resonance at a particular combination of tension and membrane thickness. As we expect, the high tension and thin membrane at the beginning of the cochlea cause it to resonate at higher frequencies (20,000 Hz), while the low tension and thicker wall at the tapered end causes it to resonate at lower frequencies (200 Hz). At the far end of the cochlea is a round window, opposite the oval window.

This may be there to prevent reflection of the wave in the basilar membrane.

Within the cochlea, on the organ of Corti, there are small cilia (hair cells) that are stimulated by these resonant waves. In one estimate, there are 30,000 nerve endings in the organ of Corti, an area about 33 mm long and 0.3 mm wide. These act as transducers, converting sound into electrical impulses. By registering which of these cilia have been stimulated (*i.e.*, at which portion of the basal membrane the wave resonated) the brain may interpret the frequencies heard quite precisely.

The exact mechanism for this is not fully understood, but some general principles are adhered to. A nerve impulse has a constant magnitude - a cell responds totally or not at all. Thus the information to the brain is encoded in the number of single events (frequency modulation) rather than the strength of a single signal (amplitude modulation). It should also be noted that the auditory nerve can also send signals to the organ of Corti. If a person stays in a noisy environment, the brain will suppress the unwanted stimuli; the sound will still register in the ear, but no signals of that frequency will be sent to the brain.

Locating Sounds

Having two ears allows a human being to determine the source of a sound. But by what mechanism is this done?

A first inclination might be to consider the loudness of the sound. Will this vary enough between the two ears to determine a source?

Consider a point source that is a horizontal distance r_o away from the forehead, at an angle q above the horizontal. If the separation of the ears is *d*, the distance from the source to the ears is then

$$r^2 = r_o^2 \pm r_o d \sin\theta + \left(\frac{d}{2}\right)^2 \approx r_o^2 \pm r_o d\theta$$

where the small angle approximation has been used. The inverse square law indicates that the change in intensity is

$$\frac{\Delta I}{I} \approx \frac{(r_o^2 + r_o d\theta) - (r_o^2 - r_o d\theta)}{r_o^2} = \frac{2d\theta}{r_o}$$

Assume: r_o: 10 m
d: 0.20 m

If our ears can distinguish a difference in loudness on the order of 0.1 dB. This will lead to an angular difference of about 33˚.

A quick classroom experiment with a tuning fork and a blindfold will show that the average person can locate a sound more precisely than this - often to within 1 - 2˚. So clearly our ears are not using loudness to determine a sound's direction.

What about the phase difference - the phase comparison of sound received in each ear? Rayleigh's Criterion for angular resolution, used primarily for optics, might work just as well here.

If I have two receivers (my ears) a distance *d* apart, and a source of wavelength l (use our typical 3000 Hz to get 11 cm),

$$\alpha = 1.22\frac{\lambda}{d}$$

The resulting angle a is about 34˚.

Again, for our preferred frequencies, there is an advantage: our head provides a "shadow" that increases the difference to about 8 dB. That improves our discernment, but not enough.

What about the time difference between the signal reaching each ear? (A similar idea is used in nuclear and particle physics detectors as "time of flight.")

If the ears are a distance d apart, we expect that

$$\theta \approx \frac{v_{sound}\Delta t}{d}$$

Here, the minimum time difference must be greater than the time for the sound to travel the cochlea and be analyzed. Treating the fluid as water (v = 1500 m/s), we get Dt = 20 μs for the sound to travel the 3 cm length. This gives us an angle of

$$\theta \approx \frac{(340\text{m/s})(20\text{ms})}{(0.20\text{m})} \approx 2^{\circ}$$

So based on our calculations, it seems reasonable that the ear uses the time difference to locate a sound source.

Hearing Aids, Cochlear Implants and Auditory Brainstem Implants

Deafness can have a number of causes; likewise the level of treatment necessary to allow a deaf patient to hear again varies. Dramatic advances have been made in the treatment of deafness over the past thirty years. In general, these can be divided into three broad categories of cause and treatment.

Outer and middle ear problems typically result in a loss of conduction to the inner ear. Hence the amplification process discussed above is impaired. (This may also occur gradually over time due to age, which can render the ossicles less flexible.) The now commonplace treatment for this is a hearing aid, which electronically amplifies the sound reaching the ear. Presently, hearing aids are very sophisticated and take advantage of both miniature electronics and filtering algorithms to filter out ambient noise.

Hearing aids will not help a person with an inner ear problem, however. In these cases, the amplified wave reaches the cochlea, but an impairment in the cilia (hair cells) in the fluid filled chamber prevent the transducing of sound waves into pulses for the auditory nerve. The current treatment for such deafness is called a cochlear implant (CI). This has been a widely used treatment with some 40,000 successful cases worldwide. The CI has an external microphone, which then transmits the sound to a receiver and microelectrode inside the inner ear. This directly stimulates the correct nerves, bypassing the malfunctioning cilia. In older patients who lost hearing (specifically, they had acquired speech prior to their hearing loss), the results have been described as miraculous.

In some cases, profound deafness occurs owing to defects after the cochlea. A very new treatment under development is an auditory brainstem implant (ABI). This next generation of treatment bypasses the cochlea and sends a signal directly to the auditory nerve. Development on this concept began in the 1970s and the current ABIs have been implanted in about 200 patients (see *Science*, 8 February 2002).

Bibliography

Burns, D.M. and S.G.G. McDonald, Physics, 2/E. Addison Wesley, Reading Mass., 1975.

Noble, J.V. "Sound, Hearing and the Human Voice," *Physics of the Human Body*. Web-based class notes, University of Virginia, 2002. http://www.phys.virginia.edu/classes/304/lects.htm

Pennycuick, C.J. *Newton Rules Biology*. Oxford University Press, Oxford, 1992.

Rauschecker, J.P. and R.V. Shannon, "Sending Sound to the Brain." *Science*, 8 February 2002.